

SEMESTER	V	QP CODE	5202
 P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA SEM V EXAMINATIONS JANUARY -2024 III B.SC. SUBJECT: MATHEMATICS SEC B			
DATE & SESSION	25.01.2024 AN	REG NO	MAX MARKS 50

SECTION-A

Answer any THREE questions of the following by choosing atleast ONE from each section . Each question carries 10 marks. 3 X 10 M = 30 M

PART-A

1. Prove that $\beta(l, m) = \frac{\Gamma(l) \Gamma(m)}{\Gamma(l+m)}$. (BT-2)
2. Prove that $\Gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$. (BT-2)
3. Find the power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$ near $x = 0$. (BT-2)

PART-B

4. Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2})$. (BT-2)
5. Prove that i) $\int_{-1}^1 P_m(x) P_n(x) dx = 0$ if $m \neq n$.
ii) $\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}$. (BT-2)
6. Show that i) $J_{-\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \cos x$ ii) $J_{\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$ (BT-3)

SECTION-B

Answer any **FOUR** questions. Each question carries 5 marks.

4 X 5 M = 20 M

7. Evaluate $\int_0^2 x(8 - x^3)^{\frac{1}{3}} dx$.

(BT-2)

8. Compute a) $\Gamma\left(\frac{-1}{2}\right)$ b) $\Gamma\left(\frac{-3}{2}\right)$.

(BT-2)

9. Show that $x = 0$ is an ordinary singular point of the equation

$$(x^2 - 1)y'' + xy' - y = 0, \text{ but } x = 1 \text{ is a regular singular point.}$$

(BT-3)

10. Find the radius of convergence of the series $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \dots$

(BT-4)

11. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.

(BT-2)

12. Evaluate $P_0(x)$, $P_1(x)$ and $P_2(x)$.

(BT-2)

13. Prove that $\frac{d}{dx}(xJ_nJ_{n+1}) = x(J_n^2 - J_{n+1}^2)$.

(BT-2)